Herbrand Property, Finite Quasi-Herbrand Models, and a Chandra-Merlin Theorem for Quantified Conjunctive Queries

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68NQRT Seminar - IRISA/INRIA December 9, 2021

# AN INTERESTING & IMPORTANT QUESTION

The Question

Why is modal logic so robustly decidable?

The Model-Theoretic Answer

Bounded-tree model property + MSOL Encoding [Vardi, 1996].

#### Few Further Answers

Encoding in:

- *two-variable fragment* of FOL (FOL[2VAR]) [Mortimer, 1975];
- guarded fragment of FOL (FOL[GF]) [Andréka et al., 1998];
- unary-negation fragment of FOL (FOL[UN]) [ten Cate and Segoufin, 2011].

### ANALYSIS OF FRAGMENTS

FOL[2VAR] is not robust, since simple extensions become undecidable.

FOL[GF] and FOL[UN] are robust and well behaved, since, as modal logic, they enjoy good model-theoretic and algorithmic properties.

	FOL[GF]	FOL[UN]
Model Checking	PTIME	PTIME NPTIME
Satisfiability	2EXPTIME-COMPLETE	2ExpTime-complete
Finite-Model Property	$\checkmark$	$\checkmark$
Craig's Interpolation	×	$\checkmark$
Beth's Definability	$\checkmark$	$\checkmark$

FOL[2VAR], FOL[GF], and FOL[UN] are orthogonal *w.r.t.* expressiveness.

# ANOTHER SIGNIFICANT QUESTION

#### The Question

Why are extensions of modal logic, as ATL\* and SL[1G], decidable?

#### The High-Level Answer

Bounded-tree model property + MSOL Encoding [Schewe, 2008, M., Murano, Perelli, Vardi, 2012].

### A Specific Problem

Why do they enjoy the bounded-tree model property in the first place?

# A FIRST-ORDER ENCODING

Consider the ATL<sup>\*</sup> formula  $[[a, b, c]] \neg \psi$  over a game with the four agents *a*, *b*, *c*, and *d*, where  $\psi$  is an LTL formula.

It asserts that agent *d* has a strategy, which depends upon those chosen by the other three ones, ensuring that the property  $\psi$  does not hold.

We can encode this property by means of the FOL sentence  $\forall a \forall b \forall c \exists d \neg R_{\psi}(a, b, c, d)$ , where  $R_{\psi}$  is the characteristic relation for  $\psi$ , *i.e.*, it is true *iff* the strategy profile (a, b, c, d) satisfies the property.

# CLASSIC CONSTRAINT VIOLATION

The FOL sentence  $\varphi = \forall a \forall b \forall c \exists d \neg R_{\psi}(a, b, c, d) \dots$ 

- has more than two variables:  $\varphi \notin FOL[2VAR]$ ;
- quantifications are not guarded:  $\varphi \notin \text{FOL}[GF]$ ;
- negation is applied to more than one free variable:  $\varphi \notin FOL[UN]$ .

We cannot derive the good algorithmic properties of ATL\* and SL[1G] from known fragments of FOL.

 $\varphi$  has quantification prefix  $\forall^3 \exists$ , so it does not even belong to any of the decidable prefix-vocabulary classes [Börger et al., 1997].

# OUR CONTRIBUTIONS

- 1 Introduction of a new way to classify FOL fragments.
- Definition of a new model-theoretic property which allows to generalise the concept of Herbrand model.

The fragments are based on the binding forms admitted in a sentence, *i.e.*, on the way the arguments of a relation can be bound to a variable.

This study allows to answer:

- **3** the question about the decidability of ATL\* and SL[1G];
- ④ an open problem in database theory.

# A TALE OF LOGIC CONNECTIONS

- 1 The *Herbrand Property* of functional first-order models.
- 2 The decidability of Conjunctive Binding Logic satisfiability.
- 3 The decidability of *Quantified Conjunctive Query* containment.
- 4 A Chandra-Merlin characterisation of QCQ containment.

# A TALE OF LOGIC CONNECTIONS

### 1 The *Herbrand Property* of functional first-order models.

- \* A structure *A* enjoys the Herbrand Property if:
  - two terms (*semantically*) *equalize* over *A iff* they (*syntactically*) *unify*.
- **2** The decidability of *Conjunctive Binding Logic* satisfiability.
- 3 The decidability of Quantified Conjunctive Query containment.
- 4 A Chandra-Merlin characterisation of QCQ containment.

# A TALE OF LOGIC CONNECTIONS

### **1** The *Herbrand Property* of functional first-order models.

### 2 The decidability of *Conjunctive Binding Logic* satisfiability.

- \* CBL is a fragment of First-Order Logic:
  - quantifications of *conjunctions* of formulas on the *same bindings*;
  - example:  $\forall x \exists y \forall z . R(x, y) \land (R(y, z) \lor \neg P(y, z)).$

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# A TALE OF LOGIC CONNECTIONS

- The *Herbrand Property* of functional first-order models.
- 2 The decidability of *Conjunctive Binding Logic* satisfiability.
- 3 The decidability of *Quantified Conjunctive Query* containment.
  - \* QCQs are an extension of classic Conjunctive Queries:
    - arbitrary quantifications of conjunctions of positive atoms;
    - fragment of CBL with no disjunction or negation;
    - example:  $\forall x \exists y \forall z . R(x, y) \land R(y, z) \land P(z, y, x).$

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# A TALE OF LOGIC CONNECTIONS

- **①** The *Herbrand Property* of functional first-order models.
- 2 The decidability of *Conjunctive Binding Logic* satisfiability.
- 3 The decidability of *Quantified Conjunctive Query* containment.
- 4 A Chandra-Merlin characterisation of QCQ containment.
  - \* Generalisation of CQ containment via homomorphism:
    - $\varphi_1 \models \varphi_2$  iff there is an "homomorphism" from  $\varphi_2$  to  $\varphi_1$  that complies with the quantification prefixes of both  $\varphi_1$  and  $\varphi_2$ .

Preface	Quantified Conjunctive Queries	Conjunctive Binding Logic	Conclusion
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# **Quantified Conjunctive Queries**

# QUANTIFIED CONJUNCTIVE QUERIES

### Relational fragment of first-order logic obtained by using $\forall$ , $\exists$ , and $\land$ .

#### Example

 $\forall x \exists y \forall z. (Px \land Rxy \land Ryz \land Qz)$ 

- Relational positive Herbrand logic in mathematical logic.
- *Quantified constraint satisfaction problems* in constraint satisfaction.

Conjunctive queries (CQs) are QCQs without  $\forall$ .

#### The Problem

Instance: A pair  $(\varphi_1, \varphi_2)$  of QCQs. Question:  $\varphi_1 \models \varphi_2$ ?

### Chen, Madelaine, & Martin (LICS'08 & LMCS'15)

Question: Is entailment of QCQs decidable?Answer: Yes it is, belongs to 3EXPTIME, but NPTIME-HARD!Question: Do entailment and finite entailment coincide?Answer: Open problem.Question: Is finite entailment decidable?Answer: Open problem.

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#### Bova & M. (LICS'17)

- Entailment and finite entailment of QCQs coincide.
- NPTIME-COMPLETE.

#### Observation

Positive instances of CQ Containment have small refutations!

$$\exists x . R(x, x) \models \exists y, z, w . R(y, z) \land R(z, w)$$

$$iff$$

$$R(c_x, c_x) \land \forall y, z, w . \neg R(y, z) \lor \neg R(z, w) \text{ is unsatisfiable.}$$

$$\underline{R(c_x, c_x)} \xrightarrow{R(c_x, c_x)} \neg R(y, z) \lor \neg R(z, w)$$

$$w \mapsto c_x$$

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$$\exists x \, . \, R(x,x) \models \exists y, z, w \, . \, R(y,z) \land R(z,w)$$
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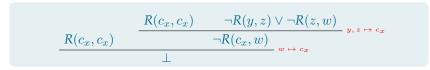


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$$\begin{array}{c|c} R(c_x,c_x) & \neg R(y,z) \lor \neg R(z,w) \\ \hline R(c_x,c_x) & \neg R(c_x,w) \\ \hline \bot & w \mapsto c_x \end{array}$$

#### Result

Positive instances of QCQ Containment have small refutations as well!

$$\exists x \forall y \, . \, R(x,y) \models \forall u, v \, \exists z \, . \, (R(z,u) \land R(z,v))$$

$$\forall y . R(c_x, y) \land \forall z . \neg R(z, c_u) \lor \neg R(z, c_v)$$
 is unsatisfiable.

$$\frac{R(c_x, y_1)}{(c_x, y_2)} \xrightarrow{R(c_x, y_1)} \frac{\neg R(z, c_u) \lor \neg R(z, c_v)}{\neg R(c_x, c_v)} \xrightarrow{y_1 \mapsto c_u; z \mapsto c_x} \frac{\neg R(c_x, c_v)}{(u_1 \mapsto c_v)}$$

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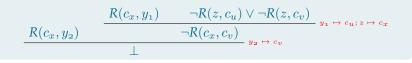
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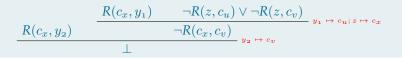


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#### Result

Positive instances of QCQ Containment have small refutations too!

$$\forall y \exists x \, . \, R(x,y) \models \forall u \, \exists z, v \, . \, (R(z,u) \land R(v,z))$$

 $\forall y . R(f_x(y), y) \land \forall z, v . \neg R(z, c_u) \lor \neg R(v, z)$  is unsatisfiable.

$$\frac{R(f_x(y_1), y_1) \quad \neg R(z, c_u) \lor \neg R(v, z)}{\neg R(y_1), y_2} \xrightarrow{y_1 \mapsto c_u; z \mapsto f_x(c_u)} \frac{\varphi_1(y_1), \varphi_2(z_u)}{\varphi_1(y_1), \varphi_2(z_u), \varphi_2(z_u)} \xrightarrow{y_1 \mapsto c_u; z \mapsto f_x(c_u)} \frac{\varphi_1(z_u), \varphi_2(z_u)}{\varphi_1(z_u), \varphi_2(z_u)} \xrightarrow{y_1 \mapsto c_u; z \mapsto f_x(z_u)} \frac{\varphi_1(z_u), \varphi_2(z_u)}{\varphi_1(z_u), \varphi_2(z_u)} \xrightarrow{\varphi_1(z_u), \varphi_2(z_u)} \frac{\varphi_1(z_u), \varphi_2(z_u)}{\varphi_1(z_u), \varphi_2(z_u)}$$

The unification  $y_1 \mapsto c_u; y_2, z \mapsto f_x(c_u); v \mapsto f_x(f_x(c_u))$  induces a Skolem homomorphism from  $\forall u \exists z.v.(R(z,u) \land R(v,z))$  to  $\forall y \exists x.R(x,y)$ .

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$$\frac{R(f_x(y_1), y_1) - R(z, c_u) \vee \neg R(v, z)}{\neg R(y_1), y_2} \xrightarrow{y_1 \mapsto c_u; z \mapsto f_x(c_u)} \frac{\varphi_1(f_x(y_1), y_1) - \varphi_2(z, c_u)}{\varphi_1(f_x(c_u)), y_1 \mapsto f_x(f_x(c_u))}$$

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### Definition

A *Skolem homomorphism* from  $\varphi_2 = \wp_2 \chi_2$  to  $\varphi_1 = \wp_1 \chi_1$  is a substitution  $\sigma$  of variables of  $\wp_2$  by terms on the vocabulary of  $\mathsf{sk}_{\wp_1}(\chi_1)$  such that:

- $lacksymbol{1}$  universal variables in  $\wp_2$  maps injectively via  $\sigma;$
- 2 every existential variable x in ℘<sub>2</sub> maps to the term σ(x) not containing images of universal variables after x in ℘<sub>2</sub>;
- **3** every atom  $R(x_1, \ldots, x_k)$  in  $\chi_2$  has an atom  $R(t_1, \ldots, t_k)$  in  $\mathsf{sk}_{\wp_1}(\chi_1)$  such that  $R(\sigma(x_1), \ldots, \sigma(x_k))$  and  $R(t_1, \ldots, t_k)$  unify.

#### Chandra-Merlin (STOC'77) - Theorem for CQ

 $\varphi_1 \models \varphi_2$  *iff* there is an homomorphism from  $\varphi_2$  to  $\varphi_1$ .

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# FINITE ENTAILMENT/CONTAINMENT OF QCQs

 $\varphi_1 \models_{fin} \varphi_2$  iff  $\varphi_2$  is satisfied on all *finite* models of  $\varphi_1$ .

Finite entailment is the relevant notion in CS applications (*e.g.*, in query optimisation the database is finite).

#### Proof Approach

We proved the collapse of entailment  $\varphi_1 \models \varphi_2$  and finite entailment  $\varphi_1 \models_{fin} \varphi_2$  of QCQs via a *domain-preserving* reduction to the satisfiability check of a CBL sentence  $\psi$ .

FOL-provers solve (in practice) the finite QCQ-entailment problem!

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#### Observations

- The results exploits the finite-model property of CBL.
- 2 It is more involved than the classic check for the satisfiability of  $\varphi_1 \wedge \neg \varphi_2$ , as the latter belongs to an undecidable fragment of FOL.
- $\psi$  unsat  $\Rightarrow \varphi_1 \models \varphi_2 \Rightarrow \varphi_1 \models_{fin} \varphi_2$
- $\psi$  sat  $\Rightarrow \mathcal{A} \models_{fin} \psi \Rightarrow \mathcal{A} \models_{fin} \varphi_1 \land \neg \varphi_2 \Rightarrow \varphi_1 \not\models_{fin} \varphi_2 \Rightarrow \varphi_1 \not\models \varphi_2$

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We proved the collapse of entailment  $\varphi_1 \models \varphi_2$  and finite entailment  $\varphi_1 \models_{fin} \varphi_2$  of QCQs via a *domain-preserving* reduction to the satisfiability check of a CBL sentence  $\psi$ .

#### Observations

- 1 The results exploits the finite-model property of CBL.
- 2 It is more involved than the classic check for the satisfiability of  $\varphi_1 \wedge \neg \varphi_2$ , as the latter belongs to an undecidable fragment of FOL.
- $\psi$  unsat  $\Rightarrow \varphi_1 \models \varphi_2 \Rightarrow \varphi_1 \models_{fin} \varphi_2$
- $\psi$  sat  $\Rightarrow \mathcal{A} \models_{fin} \psi \Rightarrow \mathcal{A} \models_{fin} \varphi_1 \land \neg \varphi_2 \Rightarrow \varphi_1 \not\models_{fin} \varphi_2 \Rightarrow \varphi_1 \not\models \varphi_2$

Preface	Quantified Conjunctive Queries	Conjunctive Binding Logic	Conclusion
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# **Conjunctive Binding Logic**

# CONJUNCTIVE BINDING LOGIC

#### CBL Syntax

Positive Boolean combinations of sentences  $\wp(\psi_1 \land \psi_2 \land \ldots \land \psi_k)$ :

- $\wp$  is an arbitrary quantification prefix;
- $\psi_i$  is a Boolean combinations of atoms over the same binding (*i.e.*, same association of variables with positions in a relation).
- $\forall x \exists y \forall z . R(x, y) \land (R(y, z) \lor \neg P(y, z)) \checkmark$
- $\forall x \exists y \forall z . R(x, y) \land (R(x, y) \lor \neg P(y, z)) \times$

CBL is incomparable with other FOL fragments (FOL[GF], FOL[UN]).

Conjunctive Binding Logic

# MAIN PROPERTIES OF CBL

#### Results for CBL

- Finite-model property.
- **2** Decidable satisfiability ( $\Sigma_3^P$ -COMPLETE).

#### Satisfiability Criterion

 $\varphi \in \text{CBL}$  is sat *iff* there is an implicant where all atoms over the same relation and unifying bindings agree on the polarity.

- $\varphi = \exists x \forall y \, . \, R(x,y) \land \neg R(y,x) \text{ is sat } \textit{iff} \, \forall y \, . \, R(c_x,y) \land \neg R(y,c_x) \text{ is sat:}$ 
  - $(c_x, y)$  and  $(y, c_x)$  unify  $\Rightarrow \varphi$  is unsat.

 $\varphi = \forall y \exists x \, . \, R(x,y) \land \neg R(y,x) \text{ is sat } \textit{iff} \, \forall y \, . \, R(f_x(y),y) \land \neg R(y,f_x(y)) \text{ is sat:}$ 

•  $(f_x(y), y)$  and  $(y, f_x(y))$  do not unify  $\Rightarrow \varphi$  is sat.

Conjunctive Binding Logic

# HERBRAND PROPERTY (I)

Two terms  $s(x_1, \ldots, x_n)$ ,  $t(x_1, \ldots, x_n)$  are *equalizable* on a structure A if  $A \models \exists x_1, \ldots, x_n \cdot s(x_1, \ldots, x_n) = t(x_1, \ldots, x_n)$ 

*Herbrand Property* of *A*: two terms *equalize* over *A iff* they *unify*.

# HERBRAND PROPERTY (II)

*Herbrand Property* of *A*: two terms *equalize* over *A iff* they *unify*.

The Satisfiability Criterion for CBL is based on the Herbrand Property:

- two unifying bindings do equalize (*i.e.*, may assume the same value) on all models;
- there is a (finite) model on which all non-unifying bindings do not equalize.
- Structures satisfying HP are called Quasi-Herbrand Models.
- Standard Herbrand Models satisfy HP.

Preface	Quantified Conjunctive Queries	Conjunctive Binding Logic	Conclusion
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# Conclusion

# SUMMING UP

- Introduction and study of a new decidable fragment of FOL.
- 2 Identification of a model-theoretic property, namely the Herbrand Property, useful to prove the decidability of fragments of first order logic with functions.
- **③** Solution of the open problem about QCQ (finite) containment.
- ④ Discover of another reason why ATL<sup>★</sup> and SL[1G] are decidable.

Thank you!

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